

Fall 2019 Math 245 Exam 1

Please read the following directions:

Please write legibly, with plenty of white space. Please fill out the box above as legibly as possible. Please fit your answers in the designated areas. To get credit, you must also show adequate work to justify your answers. If unsure, show the work. All problems are worth 5-10 points. The use of notes, calculators, or other materials on this exam is strictly prohibited. This exam will begin at 1:00 and will end at 1:50; pace yourself accordingly. Please remain quiet to ensure a good test environment for others. Good luck!

REMINDER: Use complete sentences.

Problem 1. Carefully define the following terms:

- a. prime

- b. Double Negation Theorem

- c. Conditional Interpretation Theorem

Problem 2. Carefully define the following terms:

- a. Addition Semantic Theorem

- b. Vacuous Proof Theorem

- c. contrapositive

Problem 3. Let p, q be propositions. Prove or disprove: $(p \wedge q) \rightarrow (p \rightarrow q)$ is a tautology.

Problem 4. Let $m, n \in \mathbb{Z}$. Prove or disprove: If $m|n$, then $m|n^2$.

Problem 5. Let $m, n \in \mathbb{N}_0$ with $n \geq m$. Evaluate and fully simplify $\frac{\binom{n+1}{m}}{\binom{n}{m}}$.

Problem 6. Prove or disprove: For arbitrary $x, y \in \mathbb{R}$, if x, y are both rational, then $\frac{x+y}{2}$ is rational.

Problem 7. Fix our domain to be \mathbb{R} . Simplify the proposition $\neg(\forall x \exists y \forall z, x < y \leq z)$ as much as possible (where nothing is negated).

Problem 8. State and prove modus ponens, using semantic theorems only (no truth tables).

Problem 9. Prove or disprove the proposition $\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, x^2 < y < (x + 1)^2$.

Problem 10. Let p, q be propositions. Find a compound proposition, using the operator nand (\uparrow) exactly three times (and no other operators), that is logically equivalent to $p \vee q$. Prove your answer.